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ABSTRACT This sets up relative dynamic models associated with spacecraft rendezvous, does quantitative analysis of influences associated with various types of perturbations, and derives truncation errors associated with the processes of setting up models. In conjunction with this, it give calculation examples. Results are capable of being used in the analysis of spacecraft rendezvous processes as well as error corrections.

KEY WORDS Spacecraft rendezvous Dynamic model Error analysis

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INTRODUCTION

Rendezvous and docking is one type of high level process associated with astronavigational activity. It is an aid in completing a considerable number of space missions--for example, assembly in orbit, increasing or exchanging space station compartment sections, experimental instruments and equipment, as well as other useful loads; recovering products, adding fuel, supplementing provisions, materiel, and raw materials; space maintenance and rescue missions, and so on.

Rendezvous refers to two or more spacecraft meeting each other in orbit in accordance with predetermined locations and times. Docking refers to two spacecraft structurally connecting into one body after meeting each other in orbit. Speaking from the viewpoint of flight dynamics and control, rendezvous and docking belongs to the category of spacecraft orbital control and attitude control. In theoretical terms as well as in terms of technological realization, it is quite a complicated process in both cases. As a result, when setting up dynamic models for spacecraft rendezvous, it is not only required that they be capable of comparatively accurately scribing out the relative motion processes of spacecraft but also that they be convenient for analyses and solutions. Traditional methods for setting up models often go toward two types of extremes. One type is making use of classical flight dynamics principles to obtain simple formulae, yet it is still difficult to describe spacecraft motions. The other type is to consider the influences of multiple kinds of factors. It is still difficult to have analytical results. It is only possible to rely on numerical value calculations. This article opts for the use of traditional methods for setting up models, quantitatively analyzing the influences of atmospheric resistance, solar light pressure, global deviations from a true sphere, and the gravitation of sun and moon. After that, it derives truncation errors associated with the process of setting up models, orbital errors, and so on, convenient to estimating model accuracies, offering a possibility for precision positioning.

1 THE SETTING UP OF RELATIVE MOTION EQUATIONS FOR RENDEZVOUS

Assuming that the target spacecraft is moving in the near circular orbit I, the target orbital coordinate system OXYZ is adopted as the dynamic coordinate system. Its origin point O and the center of mass of the target spacecraft are firmly connected. In conjunction with this, it follows its motion along orbit I. The X axis is within the orbital plane. In conjunction with this, it is opposite to the direction of the velocity of motion. The directions of the Y axis and the vector $\vec{r}_1$ associated with the target spacecraft center of mass in inertial space are the same, as shown in Fig.1.

In addition, assume that the pursuing spacecraft center of

mass vector in inertial space is $\bar{r}_1$, . From Newtonian laws

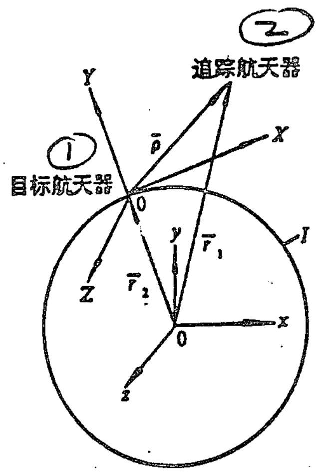

Fig.1 Orbital Coordinate System

Key: (1) Target Spacecraft (2) Pursuing Spacecraft

as well as dynamics relationships, one gets /139

$$\frac{\delta^2\bar{\rho}}{\delta t^2}\ 2\left(\bar{\omega}\times\frac{\delta\bar{\rho}}{\delta t}\right)+\bar{\omega}\times(\bar{\omega}\times\bar{\rho})+\bar{\omega}\times\bar{\rho}=\mu\frac{\bar{r}_2-\left(\frac{r_2}{r_1}\right)^3\bar{r}_1}{r_2^3}+\frac{\bar{T}}{m} \quad (1)$$

In the equation, $\bar{\rho}=\bar{r}_1-\bar{r}_2$. Due to the fact that the following spacecraft is moving in the vicinity of the target spacecraft, in comparison with $\bar{r}_1$, $\bar{r}_2$, $\bar{\rho}$ is a small quantity. Moreover,

$$\left(\frac{r_2}{r_1}\right)^3=\left[1+2\frac{y}{r_2}+\left(\frac{\rho}{r_2^2}\right)^2\right]^{-\frac{3}{2}} \quad (2)$$

As far as equation (2) is concerned, expanding in accordance with the series, equation (1) is projected toward coordinate system OXYZ, and one then obtains

$$\begin{cases} \ddot{x} - 2\omega\dot{y} = T_x / m \\ \ddot{y} + 2\omega\dot{x} - 3\omega^2 y = T_y / m \\ \ddot{z} + \omega^2 z = T_z / m \end{cases} \tag{3}$$

The equation set in question will--within a certain range--describe the relative motions of target spacecraft and pursuing spacecraft.

2 MODEL ERROR ANALYSIS

(1) The Influences of Various Types of Perturbations

During the process of deriving equation (3), spacecraft are handled as problems where they only act as two bodies and have not involved all such influences as atmospheric resistance, solar light pressure, the gravitational forces of the sun and moon, as well as global deviations from a true sphere, and so on. Moreover, as far as spacecraft carrying out rendezvous are concerned, they are generally low orbit spacecraft. Densities and velocities are both comparatively large. Therefore, atmospheric resistance, among perturbation quantities, occupies a relatively large part. It will consume most of the spacecraft energy.

When spacecraft are located outside the shadow of the earth, they undergo the effects of solar radiation pressure. Light pressure direction is from the sun pointing toward the spacecraft. Earth shadow effects in light pressure perturbations are capable of giving rise to abrupt changes in orbital altitude. The order of magnitude can reach several tens of kilometers. Besides this, the gravitational forces of the sun and moon will also produce a definite influence on relative motions of spacecraft. The order of magnitude is 103 smaller than the influence of the shape of the earth.

(2) Circular Orbit Approximations

During the process of derivation, elliptical orbits are taken as approximations of circular orbits, ignoring the influence of $\dot{\omega}$ and the influence of rates of flattening in series expansions. On the basis of the maximum values of $|\dot{\omega}|$ and $\left|\omega^2 - \frac{\mu}{r_2^3}\right|$ --through numerical value methods--it is

to calculate the influence of degrees of orbital non-roundness.

(3) Model Approximation Treatment Errors

Perturbation errors achieve correction by the introduction of perturbation forces into control quantities. In this section, we discuss the errors which approximation treatments bring with them in the process of deriving formulae.

((1)) In the derivation process, take

$$\left(\frac{r_2}{r_1}\right)^3=\left[1+2\frac{y}{r_2}+\left(\frac{\rho}{r_2}\right)^2\right]^{-\frac{3}{2}}$$

and approximate it as

$$(r_2/r_1)^3=1-3(y/r_2) \tag{4}$$

The truncation error $R_x(\rho)$ can be calculated as being

$$|R_x(\rho)|\leqslant(M/2)|\rho|^2 \tag{5}$$

In the equations,

$$M\geqslant\max|f^2(\rho)| \tag{6}$$

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In equation (6),

$$f=\left[\frac{r_2}{r_1}\right]^3=\left[1+2\frac{y}{r_2}+\left(\frac{\rho}{r_2}\right)^2\right]^{-\frac{9}{2}}$$

Therefore,

$$f^{(2)}(\rho)=-3\left\{\frac{1}{r_2^2}\left[1+2\frac{y}{r_2}+\left(\frac{\rho}{r_2}\right)^2\right]^{-\frac{5}{2}}-5\left[1-2\frac{y}{r_2}-\left(\frac{\rho}{r_2}\right)^2\right]^{-\frac{7}{2}}\cdot\left(\frac{\rho+r_2}{r_2^2}\right)^2\right\}$$

Due to y << r2, ρ << r2 , and after omitting high order terms, one has

$$|f^{(2)}(\rho)|=\left|\frac{3}{r_2^2}\left(1+2\frac{y}{r_2}\right)^{-\frac{5}{2}}\right|\ll\left|\frac{3}{r_2^2}\left(1+2\frac{\rho}{r_2}\right)^{-\frac{5}{2}}\right| \tag{7}$$

Finally, one obtains $$|R_x(\rho)| \leqslant \frac{3}{2}\frac{\rho^2}{r_2^2}\left(1+2\frac{\rho}{r_2}\right)^{-\frac{5}{2}}$$

or

$$|R_x(\rho)|_{max} = \frac{3}{2}\frac{\rho^2}{r_2^2}\left(1+2\frac{\rho}{r_2}\right)^{-\frac{5}{2}}$$

((2)) Orbital Errors Produced by Rx (ρ)

Assuming T=0, from equation (1), one has

$$\frac{\delta^2\overline{\rho}}{\delta t^2} + 2\left(\overline{\omega}\times\frac{\delta\overline{\rho}}{\delta t}\right) + \overline{\omega}\times(\overline{\omega}\times\overline{\rho}) + \overline{\omega}\times\overline{\rho} = \mu\frac{\left(\overline{r}_2 - \left(\frac{r_2}{r_1}\right)^3\overline{r}_1\right)}{r_2^3} \tag{8}$$

With regard to $$\left(\frac{r_2}{r_1}\right)^3 = \left[1+2\frac{y}{r_2}+\left(\frac{\rho}{r_2}\right)^2\right]^{-\frac{3}{2}}$$

it is possible--after considering error Rx (ρ) --to expand into

$$\left(\frac{r_2}{r_1}\right)^3 = 1 - 3\frac{y}{r_2} + R_x(\rho)$$

Equation (8) can be written to become

$$\frac{\delta^2\overline{\rho}}{\delta t^2} + 2\left(\overline{\omega}\times\frac{\delta\overline{\rho}}{\delta t}\right) + \overline{\omega}\times(\overline{\omega}\times\overline{\rho}) + \overline{\omega}\times\overline{\rho} = \mu\frac{\overline{r}_2 - \left(1-3\frac{y}{r_2}\right)\overline{r}_1}{r_2^3} - \mu\frac{R_x(\rho)}{r_2^3}\overline{r}_1 \tag{9}$$

Comparing the equation above and equation (8), it is possible to see that the error below exists

$$-\mu\frac{R_x(\rho)}{r_2^3}\overline{r}_2 = a_x\overline{X}_0 + a_y\overline{Y}_0 + a_z\overline{Z}_0$$

As far as conclusions when direct application is made of existing control forces are concerned, the orbital coordinates are then

$$
\begin{cases}
x' = \left(x_0 + \dfrac{2\dot{y}_0}{\omega} + \dfrac{4a_x}{\omega^2} \right) + 2\left(\dfrac{2\dot{x}_0}{\omega} - 3y_0 - \dfrac{a_y}{\omega^2} \right)\sin(\omega t) - 2\left(\dfrac{\dot{y}_0}{\omega} + \dfrac{2a_x}{\omega^2} \right)\cos(\omega t) \\
\qquad - \left(3\dot{x}_0 - 6\omega y_0 - \dfrac{2a_y}{\omega} \right)t - \dfrac{3a_x}{2}t^2 \\
y' = \left(4y_0 - \dfrac{2\dot{x}_0}{\omega} + \dfrac{a_y}{\omega^2} \right) + \left(\dfrac{\dot{y}_0}{\omega} - 2\dfrac{a_x}{\omega} \right)\sin(\omega t) - 2\dfrac{a_x}{\omega}t - \left(3y_0 - 2\dfrac{\dot{x}_0}{\omega} + \dfrac{a_y}{\omega^2} \right)\cos(\omega t) \\
z' = \dfrac{\dot{z}_0}{\omega}\sin(\omega t) + \left(Z_0 - \dfrac{a_z}{\omega^2} \right)\cos(\omega t) + \dfrac{a_z}{\omega^2}
\end{cases}
$$

Assume $\Delta x = x' - x,\ \Lambda v = y' - y,\ \Delta z = z' - z$ /141

In this, x, y, and z are the results when there are no control forces in the first section.

Thus, one obtains the error formulae as

$$
\begin{cases}
\Delta x = \dfrac{4a_x}{\omega^2} - 2\dfrac{a_y}{\omega^2}\sin(\omega t) - \dfrac{4a_x}{\omega^2}\cos(\omega t) + \dfrac{2a_y}{\omega}t - \dfrac{3a_x}{2}t^2 \\
\Delta y = \dfrac{a_y}{\omega^2} - 2\dfrac{a_x}{\omega^2}\sin(\omega t) - \dfrac{a_y}{\omega^2}\cos(\omega t) - 2\dfrac{a_x}{\omega}t \\
\Delta z = \dfrac{a_z}{\omega^2}\cos(\omega t) + \dfrac{a_z}{\omega^2}
\end{cases}
\tag{10}
$$

3 CALCULATION EXAMPLES AND CONCLUSIONS

Use is made of the results derived in this article to analyze orbital errors which a certain spacecraft is capable of producing. Assume that the orbital parameters of the pursuing spacecraft are: major hemiaxis a = 6 706.221 km, eccentricity e = 0.024 578, orbital angle of inclination i = 65°, ascending node celestial diameter Ω = 30°.

When the relative distances of the two spacecraft are 500km and 100km, truncation errors $Rx(\rho)$ are, respectively, 8.34m and 0.31m. In the first type of situation, the orbital error given rise to is

$$\Delta x = 80\,883.84[\omega t - \sin(\omega t)];\quad \Delta y = 40\,441.92[1 - \cos(\omega t)];\quad \Delta z = 0$$

Due to there existing in Δx a term associated with a linear increase as a function of time, as a result, divergence is relatively fast. Moreover, Δy is then an oscillating term. Δz is zero from beginning to end. Fig.2 ~ Fig.4 are computer simulation results.

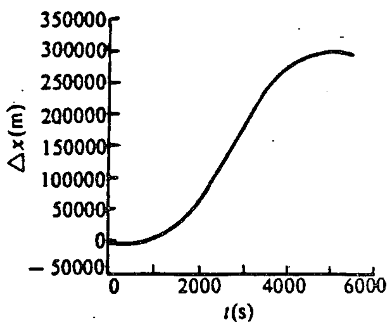

Fig.2 x Direction Error Curve

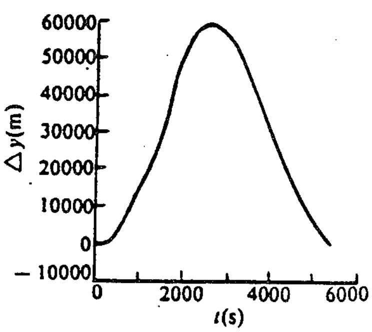

Fig.3 y Direction Error Curve

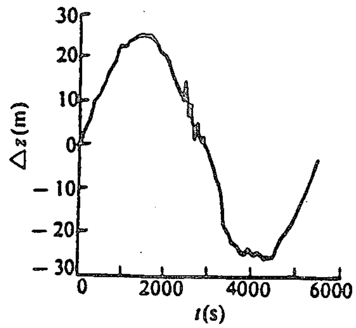

Fig.4 z Direction Error Curve

From derived results and simulation calculations, it is possible to know that, following along with the nearing of relative distances, truncation errors will tend toward zero. However, due to the oscillating nature of orbital errors--following along with enlargements of time periods--overall errors will abruptly increase. As a result, based on dynamic models associated with analytical methods, it is only possible to describe spacecraft movements within a certain range and time period. Carrying track determination precision a step further is normally completed through opting for the use of ground equipment remote control telemetry methods. The authors explored the possibility of making use of GPS (global positioning system) in combination with this article's models to carry out autonomous track determination [3].

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